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A New Approach for Selecting a Constraint in Linear Programming Problems to Identify the Redundant Constraints.

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Abstract— Linear programming (LP) is one of the most important techniques used in modeling and solving practical optimisation problems that arise in industry, commerce and management. It is well known that, for largest LP problems, only a relatively small percentage of constraints are binding at the optimal solution. In fact, large LP problems almost contain a significant number of redundant constraints and variables. Therefore it is worthwhile to devote some efforts in presolving for considerable reduction in the size of the problem. This paper presents a new approach for selecting a constraint in linear programming problems to identify the redundant constraints. The algorithm is coded by using a computer programming language C. The computational results are presented and analyzed in this paper.

Index Terms— linear programming, restrictive constraint, redundant constraints,

1 INTRODUCTION

Linear programming represents a mathematical model for solving numerous practical industrial problems such as the optimal allocation of resources. The general linear programming model with bounded variables can be stated as

$$\text{LP: Max } Z = CX$$
$$\text{Subject to } AX \le b, \qquad (1)$$
$$0 \le X \le U$$

Where X is an n x 1 vector of variables. A is an m x n matrix $[a_{ij}]$ with 1 x n row vectors A_i, i = 1,2,3,...,m, b an m x 1 vector, C an 1 x n vector and 0 an n x 1 vector of zeros . U is an n x 1 vector.

Let $A_iX \le b_i$ be the i^{th} constraint of the system (1) and let $S = \{X∈R^n / A_iX \le b_i,\ X \ge 0\}$ be the feasible region associated with system (1). Let $S_k = \{X∈R^n / A_iX \le b_i,\ X \ge 0,\ i \ne k\}$ be the feasible region associated with the system of equations $A_iX \le b_i$, i = 1, 2,3,…, m, i≠k.

The k^{th} constraint $A_kX \le b_k$, is redundant if and only if $S_k = S$ and necessary if and only if $S_k \ne S$. Many Researchers [1 - 2] and [4-17] have proposed different methods to identify the redundancies in linear programming problems. In 1989, Caron et. al [7] proposed a theorem to identify the redundant constraints, which states that the k^{th} constraint $A_kX \le b_k$ is redundant if and only if the problem LP_k has an optimal solution X^* with $A_kX^* \le b_k$, where LP_k is given by

$$LP_k : \text{maximize } A_kX$$
$$\text{Subject to } A_iX \le b_i,\ i = 1,2,3,...,m,\ i \ne k$$
$$X \ge 0.$$

Ilya Ioslovich [11] suggested an approach to identify the redundant constraints in the system of equation (1) by using a constraint instead of using all the remaining (m-1) constraints. This constraint is said to be most restrictive constraint. In this approach first the most restrictive constraint $l = \arg\min_i Z_i$ selected from the constraint set.

Where Z_i is the optimal value of LP_i. Where LP_i is

$$LP_i : \ \max Z_i = CX$$
$$\text{Subject to } A_iX \le b_i \qquad (2)$$
$$0 \le X \le U$$

Then identified the constraints $A_kX \le b_k$, is redundant if $\alpha_k^l < b_k$ Where α_k^l is the optimal value of LP_k^l. Where LP_k^l is

$$LP_k^l: \text{Maximize } \alpha_k^l = A_kX$$
$$\text{Subject to } A_lX \le b_l \qquad (3)$$
$$0 \le X \le U$$

In the Ioslovich [11] approach the most restrictive constraint has been chosen by using the optimal values Z_i, i = 1,2,3,...,m. Hence this approach consumes more number of computational efforts and time. To overcome this difficulty this paper suggests a new approach to select a restrictive constraint. Which is presented in the section 2. The section 3 illustrates the new approach with some numerical examples .The efficiency of the introduced approach is reported through various sizes of LP problems in the section 4. The section 5 draws the conclusion of the paper.

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2 PROPOSED APPROACH

In this section, a new approach is suggested to select the most restrictive constraint. The steps of the proposed approach are as follows.
Let us consider the following problem

$$\max Z = \sum_{j=1}^{n} c_j x_j$$

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$, i = 1,2,3,...,m

$0 \leq x_j \leq u_j$, j = 1,2,3,...,n

Step:1
Express each constraint with the following form by dividing each constraint by the corresponding right hand side value.

$$\sum_{j=1}^{n} \overline{a_{ij}}\, x_j \leq 1, I = 1,2,3,\dots,m.$$

where $\overline{a_{ij}} = a_{ij} / b_i$, $(b_i > 0 , \forall_i)$

Step:2

Compute $S_i = \sum_{j=1}^{n} \left|\overline{a_{ij}}\right|$ for each i∈I, I= {1,2,3,...,m}

Step:3
Select a most restrictive constraint .Where $l = \arg\max_i (S_i)$

3 Numerical Examples

This section illustrates the proposed approach and also shows the advantages of the proposed approach by solving various size LP problems

Example 1:

Consider the following LPP

Max $Z = 40x_1 + 100x_2$
Subject to
$10x_1 + 5x_2 \leq 250$
$2x_1 + 5x_2 \leq 100$
$2x_1 + 3x_2 \leq 90$
$0 \leq x_1 \leq 25$
$0 \leq x_2 \leq 20$

Solution:

Here $C = (40 \quad 100)$

$$A = \begin{pmatrix} 10 & 5 \\ 2 & 5 \\ 2 & 3 \end{pmatrix}$$

$$b^T = 250 \quad 100 \quad 90$$

$$U^T = (25 \quad 20)$$

Step 2:
$S_1 = 0.06$
$S_2 = 0.07$
$S_3 = 0.06$

Step 3
$l = \arg\max_i (S_i)$, i = 1, 2, 3.
$l = 2$

Solving the problems LP_1^2 and LP_3^2

LP_1^2 : max $\alpha_1^2 = 10x_1 + 5x_2$
Subject to
$2x_1 + 5x_2 \leq 100$
$0 \leq x_1 \leq 25$
$0 \leq x_2 \leq 20$

and LP_3^2 : max $\alpha_3^2 = 2x_1 + 3x_2$
Subject to
$2x_1 + 5x_2 \leq 100$
$0 \leq x_1 \leq 25$
$0 \leq x_2 \leq 20$

We have $\alpha_1^2 = 300$, $\alpha_3^2 = 80$
Since α_1^2 is not less than 250, α_3^2 is less than 90, constraint 3 is redundant.

Example 2:

Consider the following LPP

Max $Z = 5x_1 + 6x_2 + 3x_3$
Subject to
$5x_1 + 5x_2 + 3x_3 \leq 50$
$2x_1 + 2x_2 + x_3 \leq 40$
$7x_1 + 6x_2 + 3x_3 \leq 30$
$5x_1 + 5x_2 + 5x_3 \leq 35$
$12x_1 + 6x_2 + 9x_3 \leq 90$
$4x_1 + 1x_2 + 2x_3 \leq 20$
$0 \leq x_1 \leq 4.285$
$0 \leq x_2 \leq 5$
$0 \leq x_3 \leq 7$

Solution:

Here $C = (5 \ 6 \ 3)$

$$A = \begin{pmatrix} 5 & 5 & 3 \\ 2 & 2 & 1 \\ 7 & 6 & 3 \\ 5 & 5 & 5 \\ 12 & 6 & 9 \\ 4 & 1 & 2 \end{pmatrix}$$

$b^T = (50 \ 40 \ 30 \ 35 \ 90 \ 20)$
$U^T = (4.285 \ 5 \ 7)$

Step 2:
$S_1 = 0.26$
$S_2 = 0.125$
$S_3 = 0.533$
$S_4 = 0.4285$
$S_5 = 0.3$
$S_6 = 0.35$

Step 3

$l = \arg\max_i (S_i)$, i = 1, 2, 3,…,6

$l = 3$

Solving the problems LP_1^3, LP_2^3, LP_4^3, LP_5^3 and LP_6^3.

We have $\alpha_1^3 = 28.50$

$\alpha_2^3 = 10.00$

$\alpha_4^3 = 42.50$

$\alpha_5^3 = 78.43$

$\alpha_6^3 = 19.14$

Since $\alpha_1^3 < b_1$, $\alpha_2^3 < b_2$ $\alpha_5^3 < b_5$, $\alpha_6^3 < b_6$, constraints 1,2,5,6 are redundant.

Example 3:

Consider the following LPP

Max Z = $61x_1 + 209x_2 + 324x_3 + 33x_4 + 276x_5 + 285x_6 + 250x_7 + 100x_8 + 12x_9 + 282x_{10}$

Subject to constraints

$16x_1 + 25x_2 + 22x_3 + 4x_4 + 9x_5 + 8x_6 + 11x_7 + 29x_8 + 20x_9 + 22x_{10} \le 18$

$5x_1 + 22x_2 + 15x_3 + 30x_4 + 24x_5 + 15x_6 + 14x_7 + 28x_8 + 31x_9 + 25x_{10} \le 53$

$22x_1 + 17x_2 + 9x_3 + 32x_4 + 26x_5 + 20x_6 + 16x_7 + 16x_8 + 26x_9 + 24x_{10} \le 50$

$14x_1 + 9x_2 + 32x_3 + 22x_4 + 30x_5 + 18x_6 + 18x_7 + 32x_8 + 15x_9 + 1x_{10} \le 40$

$32x_1 + 30x_2 + 10x_3 + 30x_4 + 7x_5 + 29x_6 + 15x_7 + 1x_8 + 19x_9 + 26x_{10} \le 4$

$12x_1 + 4x_2 + 30x_3 + 11x_4 + 23x_5 + 29x_6 + 8x_7 + 2x_8 + 23x_{10} \le 31$

$22x_1 + 23x_2 + 26x_3 + 13x_4 + 6x_5 + 13x_6 + 32x_7 + 11x_8 + 8x_9 + 5x_{10} \le 39$

$0 \le X \le U$, where U^T = (0.125, 0.133, 0.4, 0.133, 0.571, 0.138, 0.266, 0.62, 0.21, 0.153)

Solution:

$$C = (\ 61\ 209\ 324\ 33\ 276\ 285\ 250\ 100\ 12\ 282)$$

$$A = \begin{bmatrix} 16 & 25 & 22 & 4 & 9 & 8 & 11 & 29 & 20 & 22 \\ 5 & 22 & 15 & 30 & 24 & 15 & 14 & 28 & 31 & 25 \\ 22 & 17 & 9 & 32 & 26 & 20 & 16 & 16 & 26 & 24 \\ 14 & 9 & 32 & 22 & 30 & 18 & 18 & 32 & 15 & 1 \\ 32 & 30 & 10 & 30 & 7 & 29 & 15 & 1 & 19 & 26 \\ 12 & 4 & 30 & 11 & 23 & 29 & 8 & 2 & 0 & 23 \\ 22 & 23 & 26 & 13 & 6 & 13 & 32 & 11 & 8 & 5 \end{bmatrix}$$

b^T = (18 53 50 40 4 31 39)

Step 2

$S_1 = 9.222$

$S_2 = 3.94$

$S_3 = 4.16$

$S_4 = 4.775$

$S_5 = 49.75$

$S_6 = 4.58$

$S_7 = 4.077$

Step 3

$l = \arg\max_i (S_i) = 5$

Solving the problems LP_1^5, LP_2^5, LP_3^5, LP_4^5, LP_6^5 and LP_7^5,

we have $\alpha_1^5 = 25.42$

$\alpha_2^5 = 28.95$

$\alpha_3^5 = 22.47$

$\alpha_4^5 = 33.33$

$\alpha_6^5 = 13.14$

$\alpha_7^5 = 15.61$

Since $\alpha_2^5 < b_2$, $\alpha_3^5 < b_3$, $\alpha_4^5 < b_4$, $\alpha_6^5 < b_6$, and $\alpha_7^5 < b_7$, constraints 2,3,4,6,7 are redundant.

4 Numerical Results

The comparative results of the two approaches for identifying the redundant constraint are presented in the following tables. The tables 1 and 2 show the comparison results of small-scale and large-scale problems. Here the number of multiplications and divisions are presented. The computational time is presented in table 1 and 2 are microseconds and milliseconds respectively. Both these approaches identify the same constraints as redundant. However, the proposed method takes very less computational effort and time compared to the Ioslovich approach [11] to identify the redundant constraints in linear programming problems.

TABLE 1: COMPARISON OF TWO METHODS (Small Scale Problems)

S. NO.	Size of the Problem		Ioslovich		Proposed	
	No. of constraints	No. of Variables	No. of Multiplications/ Divisions	Time (micro seconds)	No. of multiplications/ divisions	Time (micro seconds)
1	3	2	747	201	326	179
2	3	2	815	286	326	187
3	3	2	747	203	326	184
4	3	3	2034	285	648	185
5	3	3	1895	290	786	200
6	3	4	4245	485	1360	230
7	4	3	2682	306	972	231
8	4	3	2681	295	1248	235
9	4	5	107860	643	3795	336
10	6	3	5082	460	2724	349

11	7	10	221226	8516	94146	3797

TABLE 2: COMPARISON OF TWO METHODS (Large Scale Problems)

S. NO.	Size of the Problem		Ioslovich		Proposed	
	No. of constraints	No. of Variables	No. of Multiplications/ Divisions	Time (milli seconds)	No. of multiplications/ divisions	Time (milli seconds)
1	50	500	7299004428	157638973	608250369	19837621
2	50	500	6965109510	146332149	593487546	18657901
3	50	500	6967810329	157647651	510430124	16727382
4	240	192	8011245923	930620218	774361273	2046104
5	511	210	12577042510	1230620218	922863013	64187356

5 CONCLUSION

In this paper, a new approach is used to identify the redundant constraints and compare with Ioslovich procedure. The proposed method takes less time consumption and minimum number of computational efforts in comparison with the earlier method.

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